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THE BEVERAGE ANTENNA:
A practical way of assessing the radiation pattern using off-air signals

A.G. Lyner, B.Sc., C. Eng., M.I.E.E.

Research Department, Engineering Division
THE BRITISH BROADCASTING CORPORATION

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Summary

This Report describes a method for assessing the performance of a Beverage, long wire, receiving antenna using off-air HF broadcast signals.

The method was used to assess the improvement in performance which should result from using a lower wire antenna. It indicated a 5 dB improvement in all-round rejection of off-beam interference for the antenna which was tested, as the antenna was lowered from a height of 3 m to 1 m.

Index terms: *Antenna; HF; HF propagation; measurement; radio; reception; interference*

Issued under the Authority of

Ian Childs

Head of Research Department

Research Department, Engineering Division
BRITISH BROADCASTING CORPORATION

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1. INTRODUCTION

The Beverage Antenna is a directional receiving antenna. It consists of a long horizontal wire, supported at intervals by poles with a receiver terminal at one end and a terminating resistor (to ground) at the other. The length of the wire may be from a few tens to a few hundreds of metres. One design described in the original paper[1], written in 1923, was actually laid along the ground.

The Beverage Antenna has been used both for transmitting and receiving LF, MF and HF signals and has the great advantages of cheap and convenient construction, wide bandwidth and unobtrusiveness. As a transmitting antenna, it is somewhat inefficient, but it is used by the military under certain conditions.

There are a number of Beverage Antennas at the BBC Monitoring Service's receiving site at Crowsley Park near Caversham. They point over a range of bearings. These antennas are supported at 3 m and have lengths between about 200 and 800 metres. These dimensions were chosen to be near-optimum for the reception of Vertically Polarised (VP) MF signals. A Beverage Antenna which is low compared with the wavelength of the received signal has little or no sensitivity to Horizontally Polarised (HP) signals, so its directivity pattern is well defined, with a peak on boresight. However the sensitivity of a 3 m high wire to HP HF signals becomes significant and this alters the effective directivity pattern of an antenna in an undeterminable way, because the relative proportions of HP and VP signals are usually varying continuously.

Ever since the antennas were built, their performance has been assessed as satisfactory on the grounds that it is nearly always possible to select one (or sometimes two) antenna outputs which are definitely better than the others, with respect to signal level and interference. The signals received on the Beverage antennas have permitted some very effective monitoring over the full range of frequencies required. The possibility of re-engineering the antenna system raised the question of how well the antennas were actually performing.

Some theoretical work was carried out to give an indication of the probable performance of the antennas over the range of frequencies being used. The work also suggested that, for operation at HF, a design based on a lower wire would perform better.

The Beverage Antenna does not lend itself to convenient measurement of its directivity pattern; it cannot be mounted on a turntable, it does not even lend itself to costly helicopter measurements (because of its huge effective aperture) neither can it be modelled accurately, using much shorter wavelength signals because of the problem of modelling the very significant effect of the imperfect ground. It was decided therefore to devise an alternative method of assessing the performance of an antenna, using available off-air signals.

This alternative method was used to assess the change in HF performance which would be produced by lowering the existing Beverage antennas from 3 m to 1 m.

2. OPERATION OF THE BEVERAGE ANTENNA

The antenna has three basic modes of operation.

2.1 Sensitivity of the wire to Vertically Polarised signals

The Beverage Antenna is a travelling wave antenna. Its principle of operation is shown in Fig. 1, which shows a wave, incident from the right of the

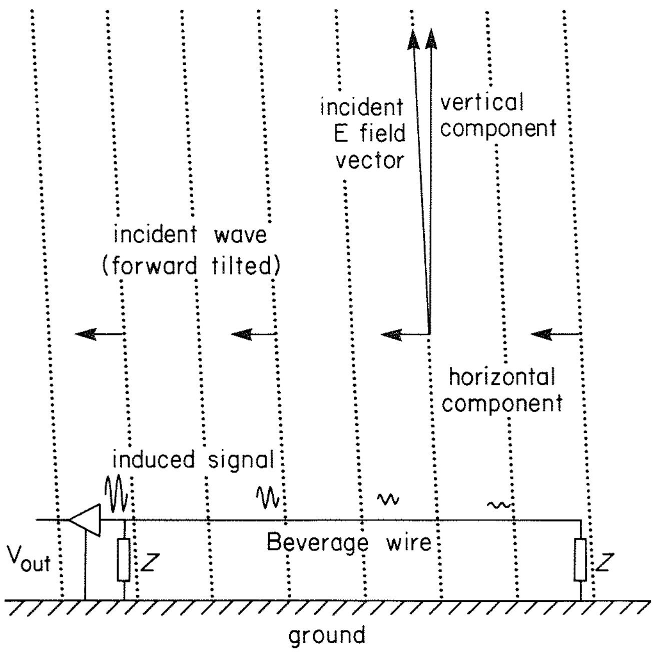

Fig. 1 - Operation of the Beverage Antenna.

figure and travelling in the parallel to the direction of the wire. This wave is shown with a forward tilt, which is present due to a combination of arrival angle from the ionosphere (in the case of a skywave) and the effect of the boundary with a lossy ground (for both skywaves and groundwaves). By virtue of this forward tilt, the wave has a small forward component of E field, in the direction of the wire.

The forward component of the wave induces a small signal into the first section of the wire. This signal travels along the wire towards the receiver. As the incident wave travels over the wire, more power is transferred to the signal on the wire. The signal voltage produced at the end of the wire consists of the (vector) sum of contributions of signals which were induced all along in the wire due to this interaction. Because the speeds of the signal along the wire and incident wave are similar, the contributions will tend to augment each other. Fig. 2 shows the effect in terms of the summation of a finite number of elements of wire. Contribution V6 is from the last element.

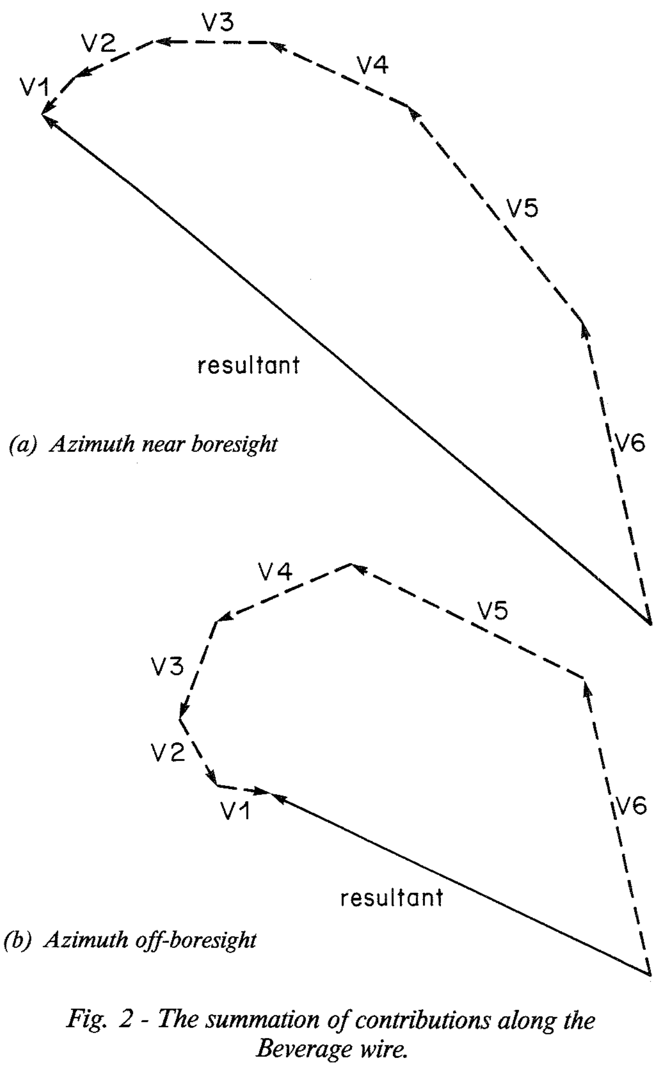

Fig. 2 - The summation of contributions along the Beverage wire.

Preceding contributions are progressively smaller, due to loss along the wire, and have a phase lag, due to the ratio of the propagation velocity of the signal on the wire and the effective speed at which incident wavefronts sweep along the wire (velocity ratio). The values of loss and transmission velocity along the wire are functions of the ground constants and the height of the wire.

As the relative azimuth of the arriving wave increases, the relative phase lag between contributions V1 - V6 will increase, producing a progressively tighter and tighter 'spiral'. The resultant will become shorter, passing through a minimum value and then going through a succession of smaller and smaller maxima. (c.f. Figs. 2(a) and 2(b).)

The purpose of the terminating resistor at each end is to dissipate signals induced by waves arriving from the 'back' of the antenna. (Of course, a single wire may be used to look in two directions at once.)

2.2 Sensitivity to Horizontally Polarised signals

The wire also has sensitivity to HP signals due to a similar mechanism. However, the sensitivity to on-axis signals is zero (the resulting E field is at right angles to the wire). Also, the effect of the ground reflections is to produce a zero of sensitivity for horizontal arrival angles. (In common with all other HP receiving antennas.) The width of the null at low elevation angles depends on the height (in wavelengths) of the wire above the ground; the lower the wire, the wider the null. Hence, an antenna which is low enough to be insensitive to HP signals at MF may have significant sensitivity at HF.

2.3 The effect of the downleads

At each end of the wire it is necessary to introduce a downlead in order to terminate the line or feed the signal to a receiver. This downlead will have a sensitivity to VP signals. The effect of the downleads can be very significant at HF, where the wire height is a relatively large fraction of a wavelength and the source impedance of the induced signals is relatively low.

There are methods for mitigating the effects of the downleads; for example, sloping downleads may be used for a high wire antenna or, for narrow-band operation, an active or passive 'counterpoise' arrangement can be used in an attempt to cancel the unwanted induced signal. The improvements tend to be effective only over a narrow bandwidth and add complication to a basically simple antenna.

2.4 Sensitivity pattern

Fig. 3 shows the theoretical sensitivity pattern of a typical Beverage antenna. The VP sensitivity pattern is shown on one side and the HP pattern is shown on the other. These figures are derived using the analysis described in the Appendix. The main feature of the VP pattern is the broad front lobe; this is due to the pick-up in the long wire. The presence of the downleads accounts for much of the fine structured side and back-lobes. The effective pattern will depend upon the actual amplitude ratio and phase relationship between the two polarisations of the incident signal. In the case of a signal which is arriving via an ionospheric path there is no way of determining the relationship; hence the pattern is indeterminable. In order to eliminate this unknown factor, it is arguably an advantage to choose antenna dimensions which suppress the HP sensitivity, even at the expense of reduced VP pick-up. (Actual signal strength is seldom a problem at HF, where external interference is more likely to be the dominant limitation.)

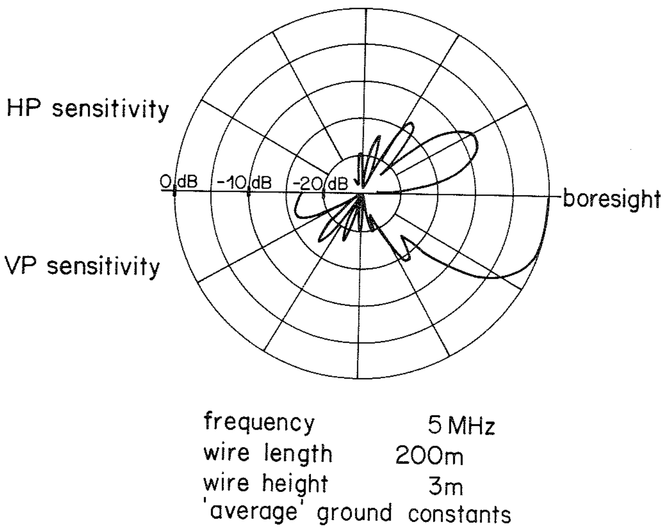

Fig. 3 - Theoretical sensitivity pattern of a Beverage Antenna.

3. A METHOD OF MEASURING PERFORMANCE

For reasons given above, conventional, formal, measurement of a Beverage antenna pattern is both uneconomic and impractical. A way has been devised, however, by which the performance of a directional antenna can be assessed. The method involves the use of a second, omnidirectional, reference antenna and some long-term and painstaking operator-intensive work.

3.1 Front-to-back ratio measurement

This is fairly straightforward and basically involves measuring the ratio between signals produced from each end of the wire by a transmission arriving on or near boresight. Because of the symmetry of the antenna, this is the equivalent to rotating the system by 180°.

Fig. 4 shows how values obtained by measurement would relate to the actual sensitivity pattern of the antenna. The signal levels of transmission *X*, (which arrives from the east of the antenna, nearly on boresight), measured on either end of the wire, will correspond directly to the values on the *actual* sensitivity pattern.

To ensure that the result is accurate, care must be taken. The transmission must be reasonably interference- and fade-free; the only way to ascertain this is for an operator to listen to the transmission during the measurement. The outputs from both ends of the antenna are fed, in turn, via an electronically controlled switch, to a measuring receiver and the results of a large number of measurements are averaged, by computer, and compared. (The output from the unused port must be terminated to ensure that the antenna is functioning correctly.)

3.2 Using off-boresight signals

The above method can be used for off-boresight signals, but, without more information, will give only a *ratio* of the sensitivity in two directions, separated by 180°. In Fig. 4, sensitivity to signals arriving from (off-boresight) transmission *Y* corresponds to the points marked '+' but, without knowledge of the level of the incident signal, only the *ratio* between the sensitivities in the two directions can be ascertained.

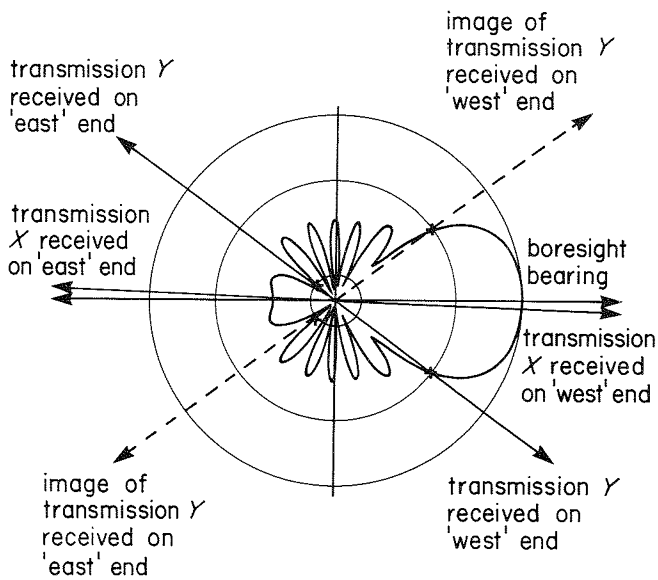

Fig. 4 - Measurement of the sensitivity pattern using off-air signals.

In order to relate the measurements, using off-boresight signals to each other *and* to the front-to-back measurements, it is necessary to know the *actual levels* of signals arriving from all directions. If these values are known, then each of the pairs of measurements, suitably normalised, may be plotted on the *same* polar graph and, ideally, should follow the radiation pattern which a conventional method would produce. An omnidirectional antenna may be used in order to obtain a measure of the levels of incident signals. If a simple, untuned, reference antenna is used then it is not actually necessary to have an absolute measurement of the incident field strength in order to normalise the results over a relatively narrow band of operation.

3.3 Use of symmetry

The sensitivity pattern of a Beverage Antenna must be assumed to be symmetrical. Any asymmetry will be due to the indeterminable ratio of amplitude and phase between the incident HP and VP waves. It is therefore valid to reflect the pattern of measured points about the line of the antenna, thus providing more points for what tends to be a sparse set of suitable measurements. (See Fig. 4.)

4. DETAILS OF THE MEASUREMENT METHOD USED

To test the above method of measuring the performance of Beverage antennas, an antenna was constructed in the grounds of Kingswood Warren.

4.1 Constructional details

The Beverage antenna was about 210 m in length and supported on a line of 3 m high wooden poles with polythene insulators at the top and at a height of 1 m above the ground. The wire was 2.24 mm copper and could be supported at either height. Despite the apparently ample area of the grounds at Kingswood Warren, there was only one possible site for an antenna of such a length. (This corresponds to one of the shortest of the Beverage Antennas at Crowsley Park.) The wire lay in a direction of 153° ETN, which meant it 'fired' in the direction of southern Europe and parts of North Africa. (Very few stations lay on the reciprocal bearing.)

4.2 Matching the transmission line

The impedance was measured with the far end short-circuited and then open-circuited. The characteristic impedance of the transmission line was taken to be the geometric mean between the two values, measured at 5 MHz. These were $(450 + j40)\Omega$ with the wire at 3 m and 390 Ω with the wire at 1 m. A simple ferrite transformer with a turns ratio of 3:1 was used to match the Beverage line to the 50 Ω feeder. The reflection coefficient with this simple matching circuit would be better than 10%, which is quite adequate to suppress the signal from 'behind', when the loss along the Beverage line is taken into consideration.

4.3 Measuring the received signals

Because of the vagaries of ionospheric propagation, it was appreciated from the start that care would be needed in the choice of signals which would be suitable for this method. There are a vast number of sources of error in the method, such as signal breakthrough, fading and interference.

It had, originally, been hoped that the system could be automated and that a suitable number of stations could be monitored automatically on a daily basis, using a computer-controlled receiver, to provide a copious enough supply of results to 'average out' variations. It was only after a number of attempts, using a range of measurement systems, that a suitable system was in fact arrived at. This final system was semi-manual and involved using a high-quality, well screened (but, alas, not computer-controllable) measuring receiver and a high quality signal selector switch, with very low cross-talk. The measurements were logged on a microcomputer. It also involved some considerable time and effort to identify stations unambiguously so that their direction of arrival could be ascertained and to listen to the transmissions constantly in order to assess their suitability.

Fig. 5 shows the basic arrangement for measuring the received signals and logging the results. The reference antenna was an active monopole, developed at Research Department, placed several metres from the Beverage antenna and from any other significant obstructions. It was assumed that its sensitivity pattern was sufficiently omnidirectional for the purposes of the measurement.

Signals from the reference antenna and from each end of the Beverage wire were fed to a three-way coaxial switch, of a type with very good isolation and which terminated the two unselected inputs with 50 Ω. A Rhöde and Schwarz measuring receiver type ESH2 was used. This receiver has exceptionally good RF screening and successfully rejected any interference generated by the nearby microcomputer. The digital value of the signal level, measured by the receiver (in dBμV), was fed to the microcomputer. Measurements were made of the signal level from each of the inputs in turn. To allow time for settling of switches, relays and ADC operation, a measurement time of 50 ms was allowed for each signal.

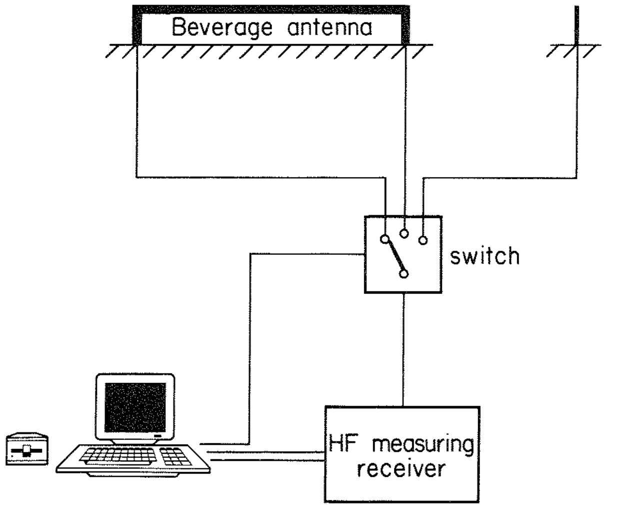

Fig. 5 - The Beverage Antenna measurement system.

4.4 Processing the measurements

1. Control and data-logging:

   One thousand triplets of interleaved measurements were made of the signal from each input port and the mean values taken. These sets of values were written to a computer diskette file for each received transmission. A header was appended to each file, containing the time of measurement, the identification of the transmission, its frequency and its bearing.

2. Data processing:

   The diskette files were all transferred to a Solbourne 4-processor workstation for further processing.

   The processing consisted, initially, of selecting files of data falling between chosen frequency limits. The thousand sets of measurements on each chosen file were averaged to produce two points on a sensitivity pattern. Because of the nature of HF reception, a number of 'wild' results were expected and the 'averaging' process took this into account.

   The mean and standard deviation of each set of a thousand values were found, and if any of the triplet of measurements lay outside the limit of ± 3 standard deviations of its particular distribution, then all three measurements were discarded. A new mean was then found for measured values of the three signals.

   Subtracting the value of reference signal level from the signal levels measured at each end of the wire, produced the normalised sensitivity values in two directions (one corresponding to the front of the pattern and one corresponding to the back of the pattern). A simple test was then performed to identify the quadrant from which the transmission arrived and to choose which end of the wire, for each transmission, corresponded to the main beam direction. This was necessary in order to be able to plot signals from both front and rear on the same axes.

5. THE RESULTS

Figs. 6 and 7 show two sets of measurements, one for each antenna height, over a range of 0— to 180° of azimuth relative to the main beam direction. These plot points were obtained over a frequency

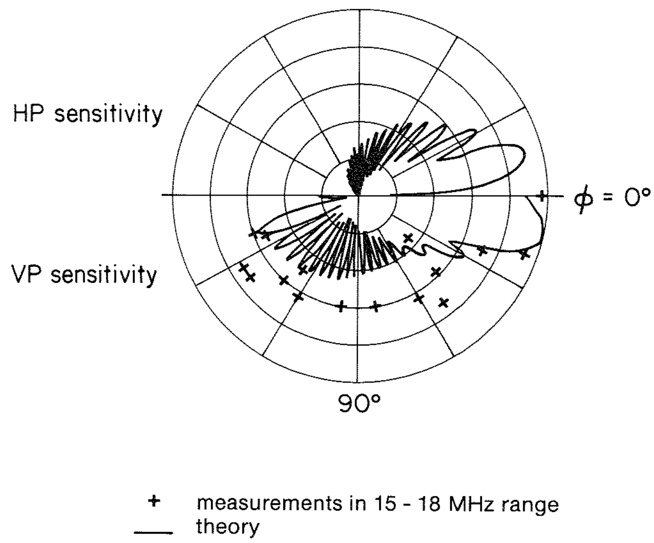

Fig. 6 - 3 m Beverage Antenna sensitivity patterns.

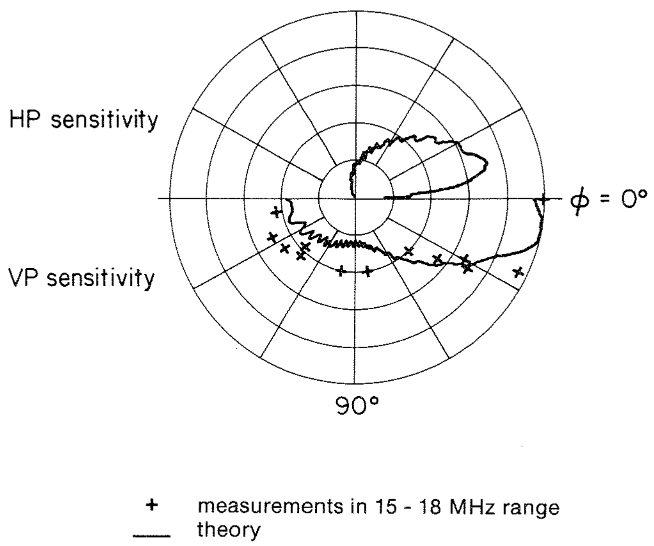

Fig. 7 - 1 m Beverage Antenna sensitivity patterns.

range of 15 to 18 MHz. (As mentioned above, because of the symmetry, measurements for each side of the antenna wire are 'folded' on to one side.)

The measured results may be compared with results of calculations for the two heights of antenna, shown in Figs. 6 and 7. Typical ground constants are used in the calculation and an arrival angle of 5° to the horizontal.

The results of the measurements show the same main features as the calculated patterns; the 'floor' value of the 3 m high antenna is about 5 dB higher than for the 1 m high antenna. There is also a peak between 40° and 50°, on the pattern of the 3 m antenna, which is probably due to the effect of HP pick-up.

5.1 Accuracy

The standard deviation of the measurements was found to be between 2 dB and 6 dB, which is not surprising in view of the nature of the propagation medium. The good agreement between the averaged values of the measurements and the theoretical patterns, justifies the large number of measurements taken in each case.

6. CONCLUSIONS

A method has been developed for estimating the sensitivity pattern of a Beverage Antenna which makes use of off-air signals which are analysed statistically by computer. The analysis software is in a portable form which could be run on any computer with reasonable performance.

The results obtained by the method agree sufficiently well with theory to justify the method.

When used to compare the performance of a high wire antenna with a low wire antenna, the method confirmed that the sensitivity pattern, at HF, is better for a low wire.

It is recommended that a similar method should be used to assess the actual performance of the antennas at Crowsley Park.

7. ACKNOWLEDGEMENT

The final success of the project relied largely on the efforts of Richard Drinkwater, who spent many hours in the measurement hut, finding, identifying and measuring enough HF signals to produce credible results and in writing the software to log the results of his measurements.

8. REFERENCES

1. BEVERAGE, H.H., RICE, C.W. and KELLOG, E.W., 1923. The wave antenna: a new type of highly directive antenna. *Trans. AIEE*, February 1923.

2. KNIGHT, P., 1971. The propagation constant of the Beverage aerial. BBC Research Department Report No. 1971/39.

APPENDIX

Derivation of the Radiation Pattern of a Beverage Antenna*

A.1. THEORY

The simplest form of Beverage Antenna is a horizontal wire several hundred metres in length, directed towards the transmitter with a vertical support at each end. The antenna is made directive by having a termination at the end nearest the transmitter and terminals at the more distant end. Others have sloping sections between the horizontal and vertical sections. (This reduces the omnidirectional effects of the vertical supports.)

A voltage is induced in the wire by virtue of the fact that the wave front of the incoming signal has a tilt (which depends on the ground parameters such as conductivity and permittivity).

Now, the open circuit voltage at the terminals of a wire antenna of arbitrary shape is given by:

$$V_{oc} = \frac{-1}{I(0)} \int_l I(l)\,\mathbf{E}\,.\,dl \qquad (1)$$

where

E is the electric field vector which would be present at a point on the wire if the wires were not there.

$I(l)$ is the current which would flow if the terminals were driven with a current $I(0)$.

Let the incoming wave make an angle ψ with the ground. If the wave is vertically polarised (i.e. the electric field is parallel to the plane of incidence) with the field strength E then, at a height of z above the ground:

$$\mathbf{E}_v(z) = \mathbf{E}\left\{e^{j\beta_o z \sin\psi} + \rho_v(\psi)\, e^{-j\beta_o z \sin\psi}\right\} \qquad (2)$$

where

β_o is the phase shift/unit length (in free space),

and

$\rho_v(\psi)$ is the reflection coefficient of a vertically polarised wave form at angle ψ.

$$\rho_v(\psi) = \frac{K_r \sin\psi - \sqrt{(K_r - \cos^2\psi)}}{K_r \sin\psi + \sqrt{(K_r - \cos^2\psi)}} \qquad (3)$$

where

K_r is the complex relative permittivity of the ground

where

$$K_r = \epsilon_r - j(1.8 \times 10^{10})\frac{\sigma}{f}$$

where

ϵ_r = relative permittivity of the ground

σ = conductivity of the ground

f = frequency of the signal.

* The analysis has been carried out by K.L. Beeke.

Similarly for a horizontally polarised wave:

$$\mathbf{E}_H(z) = \mathbf{E}\left\{e^{j\beta_o z \sin\psi} + \rho_H(\psi)\, e^{-j\beta_o z \sin\psi}\right\} \tag{4}$$

where

$\rho_H(\psi)$ is the reflection coefficient of a horizontally polarised wave.

$$\rho_H(\psi) = \frac{\sin\psi - \sqrt{(K_r - \cos^2\psi)}}{\sin\psi + \sqrt{(K_r - \cos^2\psi)}} \tag{5}$$

Equations 2 and 4 can then be used (together with Equations 3 and 5 respectively) in Equation 1 to determine the open circuit voltage induced by the received signal.

The current at distance x from the terminals is:

$$I(x) = I(0)\, e^{-\gamma H}\left[e^{-\gamma x} + \rho_i\, e^{-\gamma[2L + 2H - x]}\right] \tag{6}$$

H is the height of the wire

L is the length of the wire

ρ_i is the reflection coefficient at the terminals

γ is the propagation constant along the Beverage.

A theoretical value for the propagation constant of the Beverage Antenna is given in Ref. 2.

This gives:

$$\gamma^1 = \gamma_o\left[1 + \frac{\eta\, w}{2\pi H\, Z_c\, \gamma_o}\right]^{1/2} \tag{7}$$

where

η = intrinsic impedance of the ground.

η = $\eta_o/\sqrt{k_r}$ and $\eta_o = 120\pi\ \Omega$

Z_c = characteristic impedance of the conductor over perfectly conducting ground

= 60 log $(2H)$ (assuming the resistivity of the wire is negligible)

γ_o is the propagation constant of free space

and

$$w = \frac{-\sqrt{K_r}}{1 + \sqrt{K_r}}\left\{\frac{1}{2\sqrt{K_r}\,\gamma H} - \left[1 - \left(\frac{\gamma^1}{\gamma}\right)^2\right]^{1/2}\right\} \tag{8}$$

where

γ = propagation constant of the ground = $\gamma_o\sqrt{K_r}$

An approximate value of

$$\gamma^1 = [1 + \eta/(2\pi H Z_c\, \gamma_o)]^{1/2}$$

can be substituted into Equation 8 which can then in turn be used in Equation 7.

Note that in the following calculations it will be assumed that this propagation constant applies over its entire length. Furthermore, the Beverage Antennas currently in use at Crowsley Park do not have sloping sections (the use of sloping sections reduces the effects of the vertical sections which are omnidirectional in the horizontal plane and therefore give rise to sidelobes).

A.2. CALCULATIONS

A.2.1 Vertically polarised wave

A.2.1.1 Voltage induced on the horizontal section

The horizontal component of the vertically polarised wave of amplitude E is $E \sin \psi$. The component of $E \sin \psi$ parallel to the wire is $(E \sin \psi) \cos \phi$ (where ϕ is the azimuth angle measured from the direction of the line of the antenna). The current on the Beverage Antenna if driven is:

$$I(x) = I(0)\, e^{-\gamma H}\left[e^{-\gamma x} + \rho_i\, e^{-\gamma[2L + 2H - x]}\right]$$

Thus the voltage induced on the horizontal section of the wire is:

$$V_1 = \int_0^L e^{-\gamma H}\left[e^{-\gamma x} + \rho_i\, e^{-\gamma[2(H+L) - x]}\right] E \sin \psi \cos \phi \left[e^{j\beta_o H \sin\psi} + \rho_v(\psi)\, e^{-j\beta_o H \sin\psi}\right] e^{j\rho_o x \cos\theta}\, dx$$

$$= E \sin \psi \cos \phi \left[e^{j\beta_o H \sin \psi} + \rho_v\, e^{-j\beta_o H \sin \psi}\right] e^{-\gamma H}$$

$$\times \left\{ \frac{(e^{-(\gamma - j\beta_o \cos\theta)L} - 1)}{-(\gamma - j\beta_o \cos\theta)} + \rho_i\, e^{-2\gamma(L+H)} \frac{(e^{(\gamma + j\beta_o \cos\theta)L} - 1)}{\gamma + j\beta_o \cos\theta} \right\} \qquad (9)$$

where θ is the angle between the horizontal section and the incoming wave.

A.2.1.2 Voltage induced on the vertical sections

A.2.1.2.1 Vertical section at the terminals

The vertical component of the vertically polarised wave is $E \cos \psi$. Thus at height Z

$$E_v(z) = E \cos \psi \left\{e^{j\beta z \sin\psi} + \rho_v(\psi)\, e^{-j\beta z \sin\psi}\right\}$$

and the current drive is

$$I(z) = I(0)\left\{ e^{-\gamma z} + \rho_i\, e^{-\gamma[2(L + H) - z]}\right\}$$

Thus the voltage induced is

$$V_2 = -E \cos \psi \int_0^H \Big\{ e^{(j\beta \sin\psi - \gamma) z} + \rho_v(\psi)\, e^{-(j\beta \sin\psi + \gamma) z} + \rho_i\, e^{-2\gamma(L+2H)}\, e^{(j\beta \sin\psi + \gamma) z}$$

$$+ \rho_v(\psi)\, \rho_i\, e^{-2\gamma(L+2H)}\, e^{(-j\beta \sin\psi + \gamma) z} \Big\}\, dz$$

$$= -E \cos \psi \Big\{ \frac{(e^{(j\beta \sin\psi - \gamma)H} - 1)}{j\beta \sin\psi - \gamma} - \rho_v(\psi) \frac{(e^{-(j\beta \sin\psi + \gamma)H} - 1)}{j\beta \sin\psi + \gamma} + \rho_i\, e^{-2\gamma(L+2H)} \frac{(e^{(j\beta \sin\psi + \gamma)H} - 1)}{j\beta \sin\psi + \gamma}$$

$$+ \rho_v(\psi)\, \rho_i\, e^{-2\gamma(L+2H)} \frac{(e^{(-j\beta \sin\psi + \gamma)H} - 1)}{-j\beta \sin\psi + \gamma} \Big\} \qquad (10)$$

A.2.1.2.2 Vertical section at the termination

In this case, at height z the field strength is

$$E(z) = E \cos\psi \, e^{\,j\beta_o L \cos\theta} \left\{ e^{\,j\beta z \sin\psi} + \rho_v(\psi)\, e^{-j\beta z \sin\psi} \right\}$$

and the current when driven is

$$I(z) = I(0)\, e^{-\gamma(H+L)} \left[e^{-\gamma(H-z)} + \rho_i\, e^{-\gamma(H+z)} \right]$$

Thus the induced voltage is

$$\begin{aligned}
V_3 &= E\cos\psi\; e^{-\gamma(H+L)}\; e^{\,j\beta_o L\cos\theta} \\
&\quad \times \int_o^H \Big\{ e^{(j\beta\sin\psi+\gamma)z}\, e^{-\gamma H} + \rho_v(\psi)\, e^{-\gamma H}\, e^{(-j\beta\sin\psi+\gamma)z} + \rho_i\, e^{-\gamma H}\, e^{(j\beta\sin\psi-\gamma)z} \\
&\qquad + \rho_v(\psi)\,\rho_i\, e^{-\gamma H}\, e^{-(j\beta\sin\psi+\gamma)z} \Big\}\, dz \\
&= E\cos\psi\; e^{-\gamma(H+L)}\; e^{\,j\beta_o L\cos\theta} \\
&\quad \times \Big\{ e^{-\gamma H}\, \frac{(e^{(j\beta\sin\psi+\gamma)H}-1)}{j\beta\sin\psi+\gamma} + \rho_v(\psi)\, e^{-\gamma H}\, \frac{(e^{(-j\beta\sin\psi+\gamma)H}-1)}{\gamma-j\beta\sin\psi} + \rho_i\, e^{-\gamma H}\, \frac{(e^{(j\beta\sin\psi-\gamma)H}-1)}{j\beta\sin\psi-\gamma} \\
&\qquad + \rho_v(\psi)\,\rho_i\, e^{-\gamma H}\, \frac{(e^{-(j\beta\sin\psi+\gamma)H}-1)}{-j\beta\sin\psi-\gamma} \Big\}
\end{aligned} \qquad (11)$$

A.2.1.3 Total voltage induced by a vertically polarised wave

The voltage induced

$$V_v = V_1 + V_2 + V_3$$

where V_1, V_2, V_3 are given by Equations 9, 10 and 11 respectively.

A.2.2 Horizontally polarised wave

When the wave is horizontally polarised then no voltage is induced in the vertical sections.

The component of the field which is parallel to the wire is $E \sin\phi$. As in Section A.2.1.1 the current on the line, if driven, is

$$I(x) = e^{-\gamma H} \left[e^{-\gamma x} + \rho_i\, e^{-\gamma(2L-x)} \right] I(0)$$

and so the voltage induced is

$$V_H = E \sin\phi \, e^{-\gamma H} \int_0^L \left[e^{-\gamma x} + \rho_i \, e^{-\gamma(2L-x)} \right] \cdot \left[e^{j\beta_o H \sin\psi} + \rho_H\psi \, e^{-j\beta_o H \sin\psi} \right] e^{j\rho_o x \cos\theta}$$

$$= E \sin\phi \, e^{-\gamma H} \left(e^{j\beta_o H \sin\psi} + \rho_H\psi \, e^{-j\beta_o H \sin\psi} \right)$$

$$\times \left\{ \frac{(e^{(j\beta_o \cos\theta - \gamma)L} - 1)}{j\beta_o \cos\theta - \gamma} + \rho_i \, e^{-2\gamma L} \frac{(e^{(j\beta_o \cos\theta + \gamma)L} - 1)}{j\beta_o \cos\theta + \gamma} \right\} \qquad (12)$$

A.2.3 Mixed polarisation

If the incident wave has components parallel and perpendicular to the plane of incidence in the ratio a:b then the induced voltage is

$$V_r = aV_V + bV_H$$